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The pressure field of a spherical diffusion flame

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SUMMARY

It is usually assumed that the pressure in the neighbourhood of a diffusion flame is constant, and the momentum problem is then de-coupled from the temperature and composition fields. The observation that pressure gradient at the discontinuous flame sheet must itself be discontinuous is sufficient to motivate an examination of the true form of the pressure field, which evidently is not constant. A simple analysis of the spherical discontinuous sheet model leads to the conclusion that the structure of the intense reaction region must play an essential role in fixing local pressure variations. The amplitude of these variations is found to be uniformly $O(M^2)$, where M is a typical flow Mach number ($M \ll 1$), but significant changes within this range can take place over distances which are $O(M^{\frac{2}{3}})$ for a simple chemical reaction.

1. Introduction

A diffusion flame can be modelled as a sheet or surface which requires that all of the major field variables are continuous in its neighbourhood while certain of their derivatives must be discontinuous. This description of the flame, originally proposed by Burke and Schumann [1], is now well-recognised as a perfectly correct singular solution of the exact conservation equations in the limit of infinitely fast irreversible chemical reaction; removal of the singular behaviour through exploitation of singular-perturbation techniques, and the interesting result that flame structure can be of two main types, is also now well-understood. A recent review paper [2] discusses many of the main features of the situation and further developments have shown that, on occasions, double-sheeted solutions are necessary [3], [4]. We shall confine our present investigations to the single-sheet situation and indeed still further restrict discussion to the case of the spherical flame. This will allow us to focus on the primary task, which is to follow through the consequences of the fact that, although the *pressure* must be continuous at the flame sheet, the *pressure-gradient* must be discontinuous in the sheet model. Previous analysis of the general situation in the neighbourhood of the flame sheet has shown [5] that every component of the rate-of-deformation tensor must be continuous there; the continuity (or mass-conservation) equation then insists that density gradients must likewise be continuous. Since the sheet model is essentially one for which temperature and composition gradients are discontinuous it follows from the thermal equation of state, $p = \rho T \sum_\alpha (c_\alpha/W_\alpha)$, that

$$\left[\frac{dp}{dr}\right]_-^+ = \rho_s \left\{ \left[\frac{dT}{dr}\right]_-^+ \sum_\alpha \left(\frac{c_{\alpha s}}{W_\alpha}\right) + T_s \sum_\alpha \frac{1}{W_\alpha} \left[\frac{dc_\alpha}{dr}\right]_-^+ \right\} \tag{1}$$

for the spherical flame with dimensionless radial co-ordinate r and subscript s denoting evaluation at the flame sheet. The symbols are defined as follows; p is the pressure based on

p_c, T is the temperature based on T_c, ρ is the density based on ρ_c, c_α is the mass fraction of species α (whose molecular weight is W_α based on W_c); the square brackets [ ]$_-^+$ indicate a jump in value at the flame sheet. All subscript c quantities are constant reference values; r is measured in units of $\mathscr{D}_c/u_c$, where $\mathscr{D}_c$ is a typical mass diffusion coefficient and u_c is a typical velocity.

The conclusion of the analysis of the discontinuous sheet model is that it is inadequate to describe behaviour of the pressure near the intense reaction zone; the essential role of flame structure is discussed at the end of the paper.

So far most theoretical analyses of diffusion flames have decoupled the problem of calculating T and c_α from the momentum equation(s) by assuming that the Mach number M, namely u_c divided by the relevant sound speed,* is small; T and c_α can therefore be presumed known, together with their jump quantities if necessary. We shall find that the momentum equation, together with appropriate boundary conditions and the jump condition (1), are together sufficient to determine p and hence ρ and u, the dimensionless gas velocity based on u_c.

Only the simplest chemical kinetics, of the form

$$X + F \rightarrow P \tag{2}$$

will be considered; other idealisations will be introduced as the need arises; none affect the general conclusions of the analysis.

2. Temperature and composition fields for the sheet model

Conservation of mass insists that

$$\rho u r^2 = m_0 \tag{3}$$

where m_0 is a constant such that $4\pi m_0$ is equal to the dimensionless total mass flux from the "fuel-sphere" whose surface lies at $r = r_i$. We assume that only fuel, F, and an inert diluent species are injected through the fuel-sphere into the space $r > r_i$. We furthermore adopt the simplest solution for the diluent mass fraction c_D; it is available when all species in the mixture have the same diffusion coefficients and simply states that

$$c_D = c_{D\infty}, \quad r \geqq r_i, \tag{4}$$

where $c_{D\infty}$ is a constant. If all species have the same specific heats the communicable enthalpy h can be written as $T + Q(c_X + c_F)$, where Q (> 0) is the appropriate chemical energy. When the Lewis number is one, and all terms in the energy equation which are $O(M^2)$ are neglected, a general solution $h = H + Ce^\chi$ exists which has H and C constant, where χ is defined as

$$\chi = \int_{r_s}^r (m_0/\rho\mathscr{D}r^2)\,dr. \tag{5}$$

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* It is convenient to choose the isothermal sound speed $(p_c/\rho_c)^{\frac{1}{2}}$ in the present problem since it avoids the necessity to introduce the specific heats ratio at various points in the analysis; no confusion need arise from use of the words "Mach number" since the isothermal and the more usual isentropic sound speeds are numerically nearly the same and are identical on an order of magnitude basis.

An explanation of the foregoing results can be found in the Appendix. Here $\mathscr{D}$ is the dimensionless mass diffusion coefficient and r_s is the flame-sheet radius. Much the easiest solution has $C = 0$ and $H = T_s$, where T_s is the flame-sheet temperature; in these circumstances it is necessary to accept that T_i ($\equiv T(r = r_i)$) is determined when r_i is given, or vice versa, but this is not seriously restrictive and so we henceforth adopt the simple solution

$$T + Q(c_X + c_F) = T_s. \tag{6}$$

The fuel and oxidant-species fields for the discontinuous sheet-model are given by

$$c_F = (1 - c_{D\infty})(1 - e^\chi), \quad r_i \lessgtr r < r_s, \tag{7a}$$

$$= 0, \qquad\qquad\qquad r > r_s;$$

$$c_X = (1 - c_{D\infty})(e^\chi - 1), \quad r > r_s, \tag{7b}$$

$$= 0, \qquad\qquad\qquad r_i \lessgtr r < r_s;$$

It has been assumed that all species have the same molecular weight, namely W_c, and that no product species exists where $r \to \infty$; we now find that eq. (1) reduces to

$$\left[\frac{dp}{dr}\right]_-^+ = \rho_s \left[\frac{dT}{dr}\right]_-^+ = -\frac{2\hat{Q}m_0}{\mathscr{D}_s r_s^2}; \tag{8}$$

where

$$\hat{Q} \equiv Q(1 - c_{D\infty}). \tag{9}$$

If T_∞ is the value of T as $r \to \infty$, eq. (6) gives

$$T_s = T_\infty + \hat{Q}, \tag{10}$$

since $c_X \to 1 - c_{D\infty}$ when $r \to \infty$. The flame sheet radius is found implicitly from (7b) and this latter condition, namely

$$m_0 \int_{r_s}^\infty (dr/\rho\mathscr{D}r^2) = \ln 2. \tag{11}$$

Since $\rho\mathscr{D}$ must of necessity be about unity, (11) shows that m_0 and r_s must be of comparable size. The Mach number M has already appeared (prior to (5)) in its role as a small parameter, and we can reasonably take m_0, r_s, $\mathscr{D}_s$ and $\hat{Q}$ as $O(1)$ in the limit as $M^2 \to 0$. We reiterate that the solutions presented in this Section are correct to $O(M^2)$ relative to unity.

Finally in this Section we note that the equation of state simplifies to

$$p = \rho T \tag{12}$$

when all molecular weights are the same. From (3) it follows that

$$pu = m_0 T r^{-2}. \tag{13}$$

Since (6) and (7, a, b) give $T = T(r)$ in the intervals $r_i \lessgtr r < r_s$ and $r_s < r$ it is clear that

(13) gives pu as a known function of r in these same intervals. It is furthermore clear that pu and its first and second derivatives with respect to r are likewise known to within an error $O(M^2)$. The connection from one interval to another is made via (8) supplemented by $[p]_-^+ = 0$.

3. The momentum equation

In the present system of dimensionless variables the momentum equation for a spherically symmetrical flow is

$$\frac{dp}{dr} = M^2 \left\{ \frac{d}{dr} \left[\tfrac{4}{3} \rho \mathscr{D} Sc \left(\frac{du}{dr} - \frac{u}{r} \right) \right] + 4 \rho \mathscr{D} Sc \frac{d}{dr} \left(\frac{u}{r} \right) - \frac{m_0}{r^2} \frac{du}{dr} \right\}, \tag{14}$$

where Sc is the Schmidt number. The momentum equation (11) is discussed in the Appendix, and the Schmidt number is also defined there. Evidently u can be eliminated in favour of p by using eq. (13) but it is also necessary to make some decision about the behaviour of the product $\rho \mathscr{D}$. The diffusion coefficient varies with T and p in proportion to $T^{1+\omega}/p$; more extensive analysis has shown that nothing of the broad features of the solution is sacrificed by making the index $\omega = 0$; it follows from eq. (12) that we can set[*]

$$\rho \mathscr{D} = 1. \tag{15}$$

Then (13), (14) and (15) combine to give

$$p^3 \frac{dp}{dr} = M^2 \left\{ \tfrac{4}{3} Sc \left[-fp \frac{d^2p}{dr^2} + p^2 \frac{d^2f}{dr^2} - 2p \frac{df}{dr} \frac{dp}{dr} \right.\right.$$

$$\left. + 2f \left(\frac{dp}{dr} \right)^2 + \frac{2}{r} p^2 \frac{df}{dr} - \frac{2}{r} pf \frac{dp}{dr} - \frac{2}{r^2} p^2 f \right]$$

$$\left. - \frac{m_0}{r^2} \left[p^2 \frac{df}{dr} - pf \frac{dp}{dr} \right] \right\}, \tag{16a}$$

where

$$f = f(r) = m_0 T(r)/r^2. \tag{16b}$$

As it stands (16a) is exact; if one uses (6) and (7) to provide the function $T(r)$ then an error term of order $O(M^4)$ in the right-hand side is implied. In any event (16) is a non-linear second-order equation for p as a function of r, and requires two boundary conditions which we shall provide in the form

$$p(r_i) = p_i \text{ (given)}, \tag{17a}$$

$$p(r \to \infty) \to 1. \tag{17b}$$

Eq. (17b) implies that it is the pressure arbitrarily far away from the flame and fuel spheres that is used for the reference pressure p_c (see (1), et seq).

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[*] The case $\omega = 1$, $\rho \mathscr{D} = T$ has been treated by Cooper [6] and confirms the foregoing statement.

4. Approximate solutions for the flame sheet pressure

An exact solution of (16a) is not available, but it is clear that a constant value of p is not even acceptable locally as a solution, since substitution of this form into the equation produces the contradictory result that p' is proportional to M^2. (N.B. We use a prime from now on to denote differentation with respect to the independent variable.)

However, since M^2 is a small number it is evidently reasonable to propose approximate solutions in $r \gtrless r_s$ which begin

$$p = p(r; M^2) \sim A^{(\pm)} + M^2 p^{(\pm)}(r); \quad r \gtrless r_s. \tag{18}$$

$A^{(\pm)}$ are two constants, not necessarily the same, which are presumed to be $O(1)$ in the M^2 limit. The equations satisfied by $p^{(\pm)}$ are found by putting (18) into (16a), dividing through by M^2 and taking the limit $M^2 \to 0$. The result is, in view of (16b),

$$A^{(\pm)} p^{(\pm)\prime} = \tfrac{4}{3} Scm_0 \left(\frac{T'}{r^2} \right)' - \frac{m_0^2}{r^2} \left(\frac{T}{r^2} \right)', \tag{19}$$

and it follows at once that

$$A^{(\pm)} p^{(\pm)} = \tfrac{4}{3} Scm_0 \frac{T'}{r^2} - m_0^2 \frac{T}{r^4} - 2m_0^2 \int^r \frac{T}{r^5} dr + B^{(\pm)}, \tag{20}$$

where $B^{(\pm)}$ are constants of integration for $r \gtrless r_s$.

Results (6) and (7) show that

$$T = T_s \mp \hat{Q}(e^\chi - 1), \quad r \gtrless r_s. \tag{21}$$

In view of (15), (5) shows that

$$\chi = \frac{m_0}{r_s} - \frac{m_0}{r}, \tag{22}$$

and (11) gives

$$m_0 = r_s \ln 2. \tag{23}$$

Consequently it is possible to carry out the integration in (20) and so express $p^{(\pm)}$ as an explicit function of the radius r. In fact

$$-2m_0^2 \int^r \frac{T}{r^5} dr = \frac{m_0^2}{2r^4} [T_s \pm \hat{Q}] \pm 2 \frac{m_0^2}{r^4} \hat{Q} e^\chi N \left(\frac{r}{m_0} \right); \quad r \gtrless r_s, \tag{24a}$$

where

$$N(x) = x + 3x^2 + 6x^3 + 6x^4. \tag{24b}$$

We can take $A^{(+)} = 1$ in $r > r_s$, so that condition (17b) is met by making $p^{(+)}(r \to \infty) \to 0$. Noting that $e^\chi \to 2$ as $r \to \infty$ we find that

$$B^{(+)} = -24\hat{Q}/m_0^2. \tag{25}$$

In rather less explicit form we can also write the solution as

$$p^{(+)} = \tfrac{4}{3} Scm_0 \frac{T'}{r^2} - m_0^2 \frac{T}{r^4} + 2m_0^2 \int_r^\infty \frac{T}{r^5} \, dr; \quad r > r_s. \tag{26}$$

This version is useful for later purposes of comparison. When $r \to r_s$ ($\chi \to 0$), $p^{(+)}$ takes the value $p_s^{(+)}$, which can easily be found from (26), etc., if need be; we note that $p_s^{(+)}$ is always negative.

The solution $p \sim 1 + M^2 p^{(-)}$ appears to be satisfactory for all $r > r_s$ until we recall that continuity of p must make $p \sim 1 + M^2 p^{(-)}$ in $r < r_s$ ($p^{(-)}$ follows from (20) with $A^{(-)} = 1$); however the subsequent conclusion that $[dp/dr]_-^+$ is $O(M^2)$ is at variance with the requirement in (8), which insists that this jump must be $O(1)$.

The fact that M^2 multiplies p'', the highest derivative of p, in (16a) naturally leads one to suspect singular perturbation in the limit as $M^2 \to 0$. In this case it appears that inner, or boundary-layer-like, behaviour of the variables is to be found in the neighbourhood of the flame sheet at r_s. This is an interesting feature since one would normally anticipate singular behaviour in the vicinity of a boundary such as $r = r_i$, where the initial, outer, approximation fails to satisfy some imposed condition such as (17a). Evidently the sheet at r_s constitutes a boundary in the particular sense that an inner, boundary-layer-like, solution exists in its neighbourhood. It is unusual in that continuity or jump conditions serve to describe the behaviour of the pressure near r_s, rather than some specific statement about the value of the pressure at the sheet; indeed it is part of the problem to actually determine this quantity.

The necessity to match the solution for p in the inner region near r_s with the outer solution (26) indicates that $p - 1$ must still be $O(M^2)$. The only feasible form for the new, inner, co-ordinate is therefore

$$r = r_s + M^2 R \tag{27}$$

whence, writing

$$p \sim 1 + M^2 P^{(\pm)}(R), \tag{28}$$

it follows from (16a) that $P^{(\pm)}$ satisfies the equation

$$P^{(\pm)'} = -F_s P^{(\pm)''} \tag{29a}$$

where

$$F_s \equiv 4 Scm_0 T_s / 3 r_s^2 > 0. \tag{29b}$$

Thus

$$P^{(\pm)} = C^{(\pm)} + D^{(\pm)} e^{-F_s R}. \tag{30}$$

Matching in $r > r_s$ requires $P^{(+)}(R \to \infty)$ to tend to $p^{(+)}(r = r_s) = p_s^{(+)}$, so that

$$C^{(+)} = p_s^{(+)}. \tag{31}$$

Since matching in $r < r_s$ requires us to examine $P^{(-)}$ as $R \to -\infty$ it follows that

$$D^{(-)} = 0. \tag{32}$$

The jump $[dp/dr]_-^+$ therefore reduces to $(dP^{(+)}/dR)_{R=0}$ and condition (8) gives

$$D^{(+)} = 2\hat{Q}m_0/F_s\mathscr{D}_s r_s^2 \simeq 3\hat{Q}/2ScT_s^2. \tag{33}$$

The final result in (33) derives from (29b), (15) and (12), plus recognition of the fact that p at r_s is equal to 1 to within an error which is $O(M^2)$.

Finally, continuity of p at r_s makes

$$C^{(-)} = C^{(+)} + D^{(+)} \simeq p_s^{(+)} + 3\hat{Q}/2ScT_s^2. \tag{34}$$

We note that $1 + M^2 C^{(-)}$ is the value of the pressure p at the flame sheet; $p_s^{(+)}$ is known to be negative, so the sign of $C^{(-)}$ may be positive or negative depending on the values of $\hat{Q}$, T_s and Sc.

The solution for p in $r < r_s$ is continued by matching $p^{(-)}$ in (20) with the constant $C^{(-)}$; in view of (32) $C^{(-)}$ is not only the value of $(p - 1)/M^2$ at $r = r_s$ but is also the entire inner solution in $r < r_s$. Clearly we must make $A^{(-)} = 1$ and one readily finds that

$$p^{(-)} = \tfrac{4}{3}Scm_0 \frac{T'}{r^2} - \frac{m_0^2}{r^4} T + 2m_0^2 \int_r^{r_s} \frac{T}{r^5} dr + B^{(-)} \tag{35a}$$

where

$$B^{(-)} = C^{(-)} - \tfrac{4}{3}Scm_0^2 \frac{\hat{Q}}{r_s^4} + m_0^2 \frac{T_s}{r_s^4}. \tag{35b}$$

At this stage we have an estimate of the pressure field from $r \to \infty$ down through to values of r which lie well within the flame sheet. A more detailed analysis, together with some numerical examples, is provided in [6]. The main features of this pressure field, namely the very rapid drop in pressure which occurs near the sheet plus the fact that the amplitude of the total variation is universally small and $O(M^2)$, are interesting and we shall return to consider them again shortly. However we must recognise that (34) cannot provide the full solution for p right down to r_i, since it will not usually give the value p_i (see (17a)) at this location. The remedy is of course the insertion of another pressure boundary-layer between solution (34) and the imposed value (for details see reference [6].) Since our object is to study the pressure field near the flame we shall not pursue this matter any further.

5. The essential role of flame structure

The analysis of the discontinuous sheet model of the flame has shown that a region of rapid pressure variation exists in a domain of dimension $O(M^2)$ adjacent to the flame sheet at r_s. Referring to (16a), this conclusion has been reached on the assumption that f, f' and f'' are all $O(1)$ on either side of the sheet. However it is known (reference [2]) that the flame has a structure which makes T, T' and T'' and hence of course f, f' and f'' too, vary continuously in the neighbourhood of r_s, as well as elsewhere within the field. Most importantly, for the simple reaction introduced in (2), T'' is proportional to $\varepsilon^{-\frac{1}{3}}$ over a domain of dimension $O(\varepsilon^{\frac{1}{3}})$ centred round r_s, where ε is a small number equal to the ratio of a typical chemical reaction time to a typical diffusion time $\mathscr{D}_c/u_c^2$. It is shown in [2] that ε

is proportional to M^2, so that T'' and hence f'' is large like $M^{-\frac{2}{3}}$ over a spatial interval whose dimension is proportional to $M^{\frac{2}{3}}$.

An integral of f'' over an interval of length proportional to $M^{\frac{2}{3}}$ will yield a $O(1)$ quantity. We therefore see from (16a) that p is uniformly $1 + M^2 p^{(1)}$ throughout the region of interest, where $p^{(1)}$ is $O(1)$, and that $p^{(1)}$ can therefore be evaluated to the required order of accuracy from an equation like (19). It follows that a uniformly valid solution in $r > r_i$ can be written as

$$p^{(1)} = \tfrac{4}{3} S c m_0 \frac{T'}{r^2} - m_0^2 \frac{T}{r^4} + 2m_0^2 \int_r^\infty \frac{T}{r^5} dr, \quad r > r_i. \tag{36}$$

The formal similarity of this result with (26) is noteworthy. However the latter only applies in $r > r_s$ with T given by the upper-sign version of (21). In the present case it is necessary to recognise that (21) only holds outside a domain $|r - r_s| = O(M^{\frac{2}{3}})$ and that within this domain [2]

$$T \sim T_s - \varepsilon^{\frac{2}{3}} Q \{ \mathscr{C}_X(R_f) + \mathscr{C}_F(R_f) \} \tag{37a}$$

where

$$R_f = (r - r_s)/\varepsilon^{\frac{2}{3}}, \tag{37b}$$

since c_α is asymptotically equal to $\varepsilon^{\frac{2}{3}} \mathscr{C}_\alpha(R_f)$ with $\mathscr{C}_\alpha = O(1)$ when $R_f = O(1)$. An additive composite expansion [7] for T gives

$$T \sim T_s \mp \hat{Q}(e^\chi - 1) - \varepsilon^{\frac{2}{3}} Q \{ \mathscr{C}_X(R_f) - \mathscr{C}_X(R_f \to \pm\infty)$$
$$+ \mathscr{C}_F(R_f) - \mathscr{C}_F(R_f \to \pm\infty) \}, \quad r \gtrless r_s \tag{38}$$

and

$$T' \sim \mp \hat{Q} \frac{m_0}{r^2} e^\chi - Q \{ \mathscr{C}'_X(R_f) - \mathscr{C}'_X(R_f \to \pm\infty)$$
$$+ \mathscr{C}'_F(R_f) - \mathscr{C}'_F(R_f \to \pm\infty) \}, \quad r \gtrless r_s, \tag{39}$$

where $\mathscr{C}'_F \equiv d\mathscr{C}_F/dR_f$, etc. Consequently the second and third terms in (36) are well-enough approximated by using the upper-sign version of (21), even within the domain $r - r_s = O(M^{\frac{2}{3}})$. It is, however, vital to use (39) to estimate the first term within this latter domain; observing that $Q\mathscr{C}'_X(R_f \to +\infty)$ will cancel with the first term in (39) for $r > r_s$, while $\mathscr{C}'_F(R_f \to +\infty) \to 0$, it can be seen that T' will be zero somewhere within the intense reaction region $R_f = O(1)$ (see, for example, reference [2].) At this location the pressure will be given by

$$p^{(1)} \simeq -m_0^2 \frac{T_s}{r_s^4} + 2m_0^2 \int_{r_s}^\infty \frac{T}{r^5} dr \tag{40}$$

$$= p_s^{(+)} + \tfrac{4}{3} S c \frac{m_0^2}{r_s^4} \hat{Q},$$

where the last result follows from (26) and (21) and the error is $O(M^{\frac{2}{3}})$. Comparing this result with (34) it is clear that the magnitude of the pressure within the sheet is not properly

predicted by confining oneself to the sheet model. Certainly the physical extent of the relatively rapid pressure variations is substantially greater at $O(M^{\frac{4}{3}})$ than the $O(M^2)$ interval predicted by the sheet model (see (27) and (28)) and it is clear that the actual structure of the intense reaction region has a direct influence on the pressure variations in its vicinity.

The result that all pressure variations in the vicinity of the flame are small and $O(M^2)$ is a validation of the assumption of constant pressure on which most flame analyses are based. We observe that pressure gradients are apparently at most $O(M^{\frac{4}{3}})$ for the present reaction model.

Acknowledgements

Appendix

The necessary conservation equations, in particular those for energy and momentum, can be found in Chapter 1 of the book by Williams [8]. We note the following specialisations of these equations to meet our present needs.

With a typical dimensional specific heat at constant pressure given by C_c (a constant) we assume that the dimensionless specific enthalpy of each species in the mixture is found by dividing its dimensional value by $C_c T_v$ and that this dimensionless enthalpy h_α is then given by the simplest useful expression, namely $h_\alpha = C_\alpha T + Q_\alpha$ where C_α is the dimensionless specific heat of species α and Q_α is its dimensionless energy of formation; if we furthermore assume that all the C_α are the same we may set each $C_\alpha = 1$ without loss of generality; in addition we can make $Q_\alpha = 0$, $\alpha = X, F, D$ and $Q_P = -Q$. Since the enthalpy per unit mass of the mixture must be found from the sum of $c_\alpha h_\alpha$ taken over all α it follows that it will be given by $T + (c_X + c_F + c_D)Q - Q$. But c_D is constant (see (4)) so that we can ignore the terms $c_D Q - Q$ and use $h = T + (c_X + c_F)Q$ as the appropriate dimensionless communicable enthalpy.

The Lewis number is defined to be $Le = \rho' \mathscr{D}' C_c / \lambda'$, where ρ', $\mathscr{D}'$ and λ' are dimensional density, diffusion coefficient and thermal conductivity respectively. We assume that the Lewis number is a constant, so that $Le = \rho_c \mathscr{D}_c C_c / \lambda_c$ too (where λ_c is a constant typical value for the thermal conductivity); it follows that $\lambda' / \lambda_c = \rho \mathscr{D}$ and this fact together with the additional assumption that $Le = 1$ accounts for the solution quoted for h just above equation (5). The $O(M^2)$ terms which are neglected in the present low-speed situation are those which account for work done by the thermodynamic pressure and for the viscous dissipation effect.

The Schmidt number is $Sc = \mu' / \rho' \mathscr{D}'$ which, since it is presumed to be constant, is also equal to $\mu_c / \rho_c \mathscr{D}_c$; μ' and μ_c are dimensional and typical dynamic viscosity coefficients respectively. In forming the dimensionless version of the momentum equation (14) the quotient $\mu' / \rho_c \mathscr{D}_c$ appears in the viscous terms; from the foregoing $\mu' / \rho_c \mathscr{D}_c = Sc(\rho' \mathscr{D}' / \rho_c \mathscr{D}_c) = \rho \mathscr{D} Sc$. The momentum equation (14) then follows from a straightforward application of the results in reference [8], which apply specifically to one-dimensional flows in general orthogonal curvilinear coordinates (in our case these coordinates are of spherical polar form).

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